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Textured Time

> *The idea of the continuum seems simple to us. We have somehow lost sight of the difficulties it implies . . . We are told such a number as the square root of 2 worried Pythagoras and his school almost to exhaustion. Being used to such queer numbers from early childhood, we must be careful not to form a low idea of the mathematical intuition of these ancient sages; their worry was highly credible.*
>
> – Erwin Schroedinger

> *It was a sometime paradox, but now time has given it proof.*
>
> – Shakespeare

1.

Born to a culture of Greek and Cartesian geometrical intuitions, it is perhaps inevitable that we conceptualize time as a linear progression from earlier events to later. The contingencies of our subjective experience are as tiny beads on a temporal string, ordered – like the points of the real number line – and likewise dense perhaps, but nonetheless slavishly constrained to the trichotomy of past, present, and future, just as in the mathematician's richly packed continuum a given number is either less than, equal to, or greater than another. From ancient times to our own century, careful thinkers have detected and pondered a lurking absurdity in this conceptualization of time, and it is the thesis of this paper that only by reconceptualizing the texture of time's fabric can the paradoxes be resolved.

These paradoxes may be analyzed into two distinct but related difficulties, exemplified by Zeno's ancient Paradox of the Arrow and McTaggart's contemporary paradox of the unreality of time,[1] respectively. The first demonstrates the difficulty inherent in our analysis of the time-line into a succession of dimensionless points, and the second reveals an inconsistency in our natural ordering of these points into past, present, and future events. I submit that we are under no obligation to view time and its passage in such a way as to make it subject to these objections; that there are alternative conceptualizations which, while not in themselves meeting every objection, still clearly demonstrate the possibility that time is endowed with such a texture as to render these traditional paradoxes obsolete. I will argue that the paradoxes rest ultimately upon a notion of the continuum that is rooted in the formalizations of modern mathematics, formalizations which are in deep trouble philosophically, and to which there are philosophically desirable (even perhaps mathematically desirable) alternatives.

[1] McTaggart, 1927

2.

In the first and most notorious of the time paradoxes, the Paradox of the Arrow, Zeno of Elea[2] begins with the tacit assumption that one may speak of a state of affairs in a dimensionless *instant of time*, and he founds the ensuing argument on the intuition that no change can occur in the confines of such an instant:

> Zeno asked his contemporaries to imagine an arrow in flight. If time consists of discrete instants, then at any particular instant of time, the arrow must be at a particular location, a certain point in space. At that instant, the arrow is indistinguishable from a similar arrow at rest. But this will be true of any instant of time, Zeno argued, so how can the arrow move? Surely, if the arrow is at rest at every instant, then it is always at rest.[3]

Although Zeno, a Parmenidean monist, was arguing against an atomistic view of time (and space), the paradox is not weakened by assuming he thought of time as moderns are inclined to do, as consisting of a continuum of 'instants' such as is modelled by the points of the standard real number line.

This can be made vivid by casting the paradox in more contemporary terms. Consider the 'transporter machine' on the popular television

[2]circa 450 b.c.

[3]Devlin, 1997.

program *Star Trek*. This device[4] functions in the fictional world of that program by somehow deconstructing objects – and people in particular – and 'beaming' them to a remote location, where they are reconstructed. Its exact mode of operation is left intentionally vague, but it is generally presumed to work by transmitting the actual substance of the *transportee* via a particle/energy beam, together with the encoded information specifying how his/her substance is to be reassembled. Hosts of reasons why such a device probably could not exist, whatever its mode of operation, are ready to hand, but for our purposes it is enough to consider the necessity for such a device to render the *precise make-up at a given instant* of the person to be transported. There are two difficulties. The first is a class of problems raised by quantum indeterminacy which I wish to put aside as involving difficult theoretical questions beyond the scope of this discussion, and in any event not essential to the main issue. The second difficulty is precisely the Paradox of the Arrow, in more compelling guise. Suppose the transporter succeeds in reassembling my physical self precisely at the point of destination, down to the last follicle, corpuscle, and neuron. What is my first act upon re-materialization? It is to fall down dead, and this for many

[4]Originally conceived by the writers of the program as a means of cutting production costs (characters could be moved from point of action to point of action instantaneously, eliminating the need for shuttle footage, etc.) the transporter quickly became a defining element of the show, and intricate story lines were generated by the implications of the existence of such a device.

reasons. First, several pints of blood which had been in motion in my body are stopped *in an instant*, bringing about immediate cardiac arrest. The heart, indeed, swings very like a pendulum in its complex rhythms, and would act precisely as a pendulum does when an intervening obstacle is placed in its path. At best it would jolt and fibrillate, unable to recover its rhythm on its own. This effect, which we might term 'the removal of momentum,' would be mirrored on smaller scales all through the body, which is a vessel of countless cycles of matter in motion, from the large scale motion of blood and other fluids to the tiny scale of cellular processes.[5]

This undesirable fate of the transportee is an inevitable consequence of fixing his physical state *at an instant of time*, which necessarily leaves out any information about where an object (or the state of a system) *has been* and where it *is going* – for a human being is a very dynamical system indeed.

One objection to this analysis of the transporter is that the machine may record not just the position but the momentum of each particle, so that the information transmitted includes a complete phase-space portrait of the subject. However, this objection confuses properties of a mathematical model with the actual properties of an object. In

[5]Conceivably the most devasting effect would be on the nervous system, which particularly in the autonomous functions maintains a delicate rhythm of actions and reactions. Hiccoughing is one innocuous symptom of a disturbance in this cycle; an epileptic seizure is a more dramatic one.

mathematics one may speak of an 'instantaneous rate of change,' but mathematicians themselves are careful to note that to speak in this way is to indulge a mental fiction; the actual mechanisms of mathematical analysis appeal to a limit process or a notion of infinitessimal precisely so as to avoid the logical absurdity of change occuring in a dimensionless point.

Zeno's question remains as relevant as ever: if there is no change in a dimensionless point of time, and time itself is 'made up' of such points, then where, precisely, does change reside? Where is the motion? It seems to me that only two conclusions are possible: either motion (through space as well as time) is illusory, or there is something incoherent in the notion of picking out an 'instant' of time. To accept the first conclusion, as many philosophers have done (and as Zeno might have intended), is to do considerable violence to our naive conceptions about the world, for that things certainly *seem* to move and change through time is a universal experience across cultures and epochs. But what of the second conclusion, the one Zeno may after all have intended his listeners to infer? Here it seems that a deep reconsideration of our conceptual models of space and time is in order. But, as I will show below, this need not entail the abandonment of our most basic intuitions about the world. Indeed, our current conceptual models of space and time, which invite Zeno's old argument anew, are not only historically recent and culturally influenced, but involve mathematical abstractions and empirical assumptions which have no special claim, over and above other equally natural abstractions and assumptions, to ontological verity.

McTaggart presents us with a subtler difficulty than did Zeno. He begins in the same place, by defining an *event* as 'the contents of any position in time.'[6] He then notes two kinds of relations among such events, the relation of being either 'earlier than' or 'later than,' and that of being either 'past, present, or future.' The first relation gives what he calls the B-series of events, and he notes that this series is forever fixed, i.e., that the relations of 'earlier than' and 'later than' which events enjoy with respect to one another are unchanging. The beheading of Anne Stuart has always been 'earlier than' World War II, and always will be. In short, the B-series is a strict linear ordering not subject to flux or flow. The relations arising from an event's being 'past, present, or future,' by contrast, are changing relations, and give rise to what he terms the A-series. Although, with respect to a given event, other events have the fixed relations of being more or less past, present, or less or more future, with respect to the 'present' itself these relations constantly change. On this account, it is in (or with respect to) the A-series, and there only, that change occurs.

Armed with these definitions, McTaggart argues to the unreality of

[6]McTaggart, 1927, p.24. I believe this use of the term 'event' is unfortunate, as the term itself tends to connote change and may thereby mislead the reader as to McTaggart's meaning. 'State of affairs' might have been better, but I will keep McTaggart's terminology for simplicity's sake, hoping to forestall confusion by means of this footnote.

time in two steps. First, he appeals to the 'universally admitted' notion that without change there can be no time.[7] He then abolishes the A-series by a sophisticated *reductio ad absurdum*, and with it all actual change, and *ipso facto* the reality of time.[8] The B-series necessarily follows the A-series into oblivion, for it is an essentially *temporal* series and hence, absent actual time, cannot exist.[9]

The flaw in McTaggart's argument is at its roots, in his definitions and assumptions, and when exposed will obviate the *reductio* argument entirely. It is his 'universally admitted' assertion that there can be no time without change, and his insistence that only the A-series and not the B-series permits change, that together provide the fulcrum, the point of leverage, for the strength of his argument. If these points be conceded, he has won the

[7]ibid., p. 25.

[8]ibid., pp. 31-34.

[9]That McTaggart's paradox is still prominent in the literature, indeed a touchstone for current debate, some ninety years after publication, is a testament to the robustness of an argument which initially strikes one as implausible. The *reductio* argument abolishing the A-series, in particular, looks weak at first blush: that 'past, present, and future' are relations or properties characterizing events, that they are mutually exclusive relations or properties, but that any given event must have all three. Contradiction. The obvious riposte, that events don't have these relations or properties concurrently but in temporal succession, turns out to involve a new set of similar relations or properties, e.g., 'will be past,' 'was present,' *et cetera*, in which arise the same contradiction. And so on *ad infinitum*. I find the cage of this argument unpalatably linguistic, but nonetheless difficult to escape on that account, so it is well that I've no need to address it here.

field. But if one of these be denied, he falls. If there can be time without change, then the nature or consistency of neither the B-series nor the A-series is relevant to its existence. Similarly, if change is necessary to time, but the A-series is not necessary to change, then we may deny that the reality of time is dependent on the consistency or otherwise of the A-series.

I have no idea whether change is necessary to time (I am inclined to doubt it), but I am prepared to deny that McTaggart's A-series is necessary to change. This will of course place the burden on the B-series, and leave us with the difficulty of showing how a static set of temporal relations can give rise to change, or indeed how B-series relations can even be spoken of as 'temporal' except in some formal sense, e.g., as the anisotropic fourth dimension of the cosmologist's 'block-universe.' But at least this difficulty has the virtue of familiarity – for it is precisely the Paradox of the Arrow! Once again we look for change in the ordered, dimensionless points of a continuum whose relations are eternally fixed.

3.

To meet this challenge, it will be necessary to analyze the 'Greek and Cartesian geometrical intuitions' with which this essay began. That the modern real number line of mathematics has become the universal model for time and space wants some investigating. What is this real number line? Whence did it arise, and how is its use as the preferred model of the continuum justified? Do alternatives exist, and if so what implications do they have for the paradoxes under consideration?

In fact the 'set of real numbers,' which I will denote hereafter by **R**, has only been with us in its modern conception for about 100 years. Prior to its formalization in set-theoretic terms during the 19th century by Dedekind and others, **R** was very problematic. Its dual identity, as a model of the continuum and as the numerical field – the very stuff – of quantitative mathematics, created a rich interplay of often conflicting ideas about the nature of both space and number. It was this philosophical breathing room that made possible such ideas as Newton's *fluxions*[10] and Leibniz's *infinitessimals*, which were conceived of as quantities so small as to be less than any given magnitude but still greater than nothing; ideas long discredited and only recently (partially) rehabilitated. Even so, by the beginning of the 20th century the tension between the spatial and numerical aspects of **R** was firmly decided in favor of the numerical aspect, and this has had a profound consequence for the paradigms of modern science. It is this consequence that is the crux of this essay.

It will be necessary to briefly review the standard formalization of **R**. We begin with the integers, $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$, which I will take as given.[11] The set of rational numbers, i.e., of ratios, is then defined in the

[10] which he later renamed *evanescent increments.*

[11] The integers have their own philosophical problems (see for example Frege, 1884), but these are not important to the argument presented here.

obvious way as the collection of all ordered pairs of integers (p, q) [12] with $(p_1, q_1) < (p_2, q_2)$ if and only if $p_1 \times q_2 < p_2 \times q_1$. It was learned by the ancient Greeks that the rational numbers fail to exhaust all possible relative magnitudes; e.g., the ratio of the diagonal of a square to its side. To wit: $\sqrt{2}$ is not a rational number. As late as 1870, a sound formalization of the so called *ir*rational numbers was still elusive. It had long been known that one could 'sneak up' on irrational quantities by means of rational ones, and this is reflected in the fact that one may approximate the 'actual value' of $\sqrt{2}$, for example, by longer and longer terminating decimals (1.4, 1.41, 1.415, *et cetera*), each of which is expressible as a ratio of (larger and larger) integers. However, this is an infinite process, and classical mathematicians retained a strong aversion[13] to admitting actual infinities into their methods. Then, in 1872, a crucial conceptual shift occured, voiced first by Karl Weierstrass and subsequently made precise by Richard Dedekind.[14] Rather than taking the approximating series of rational quantities as a means of *approaching* the irrational quantity, take the described collection of rational quantities actually *to be* the irrational quantity. Dedekind took this idea further, defining $\sqrt{2}$, for example, to be the set of all rational numbers whose square is less than 2. In modern notation:

$$\sqrt{2} = \left\{ \frac{p}{q} : \left(\frac{p}{q} \right)^2 < 2 \right\},$$

[12] usually written $\frac{p}{q}$.

[13] rooted in Aristotle: see *Physics*, Book III.

[14] Boyer, p. 563.

where p and q are understood to be integers. This is now called a *Dedekind cut*, for the definition neatly *cuts* the real numbers into two halves, with $\sqrt{2}$ being defined as the 'point' of the cut. Boyer sums up Dedekind's intuition in this way:

> Upon pondering this matter, Dedekind came to the conclusion that the essence of the continuity of a line segment is not due to a vague hang-togetherness, but to an exactly opposite property: the nature of the division of the segment into two parts by a point on the segment.[15]

Gone are Newton's *fluxions* and Leibniz's *infinitessimals*, to be replaced by a continuum each of whose points is merely a division into two parts, separating that which has come before from that which comes after.

The analogy to *instants of time*, conceived of as merely the dividing points between past and future, is striking. Indeed the matter goes beyond analogy, for in all modern science time is parametrized by a real variable t, real in precisely Dedekind's sense of being a mere *cut*, a slice through the substance of the actual. Hence of necessity the Now has for us no structure, no texture; it is a dimensionless nothing between that which was and that which will be.

[15]ibid., p. 564.

4.

I concede that this model of the continuum has served both mathematics and the physical sciences very well. Like all good formalizations, it has provided the structures needed to push forward mathematical discovery, and by extension physical insight, and is thereby justified. The immense edifice of 20th century mathematics and theoretical physics is founded upon Dedekind's conceptualization, and there is no compelling empirical reason (that we can properly identify) for researchers in these sciences to abandon it. Yet, for the philosopher, Zeno's ghost lingers to haunt us, for the empirical fact that things actually *happen* stands in direct contradiction to his conclusion that instants of time, thus conceived, without structure or parts, cannot provide an arena for change. On this basis, the pragmatic justification given above for the use of our contemporary model of the continuum begins to look alarmingly weak.

But what alternative exists? Let us first note that, since the 19th century, certain results in the foundations of mathematics assert that this formalization of the real numbers is, in some important sense, *incomplete*. For instance, the Continuum Hypothesis of Georg Cantor, which asserts certain basic facts about the nature of the continuum in its modern conception, is undecidable from the established axioms of set theory.[16] Similar considerations led the great 20th century logician Kurt Gödel to

[16]This was proved jointly by Kurt Gödel and Paul Cohen; see Rucker, 1995, p. 252.

conjecture that the points of the (Dedekind/Cantor) real number line are insufficient to exhaust an absolutely continuous line, but rather form "some kind of scaffold on the line."[17] In other words, Dedekind's conceptualization somehow fails to capture the richness of structure of the continuum, providing instead a mere skeleton outline, so to speak, much the way that the map of a city, however brilliantly drawn, can never capture all of the relationships among its features. These difficulties remind us that a given formalization can never reach, conceptually, beyond what it assumes, but can only serve to clarify and make precise those very assumptions, and to reveal their logical consequences. Dedekind began with the assumption that the points on a geometrical line can be put into a one-to-one correspondence with the real numbers,[18] and implicit within that assumption was Dedekind's own (and his peers') conception of the structure of a geometrical line.

The question of an alternative remains. Although at present there does not exist an alternative *formalization* of the continuum which meets every philosophical objection, it is clear that even within the strictures of mathematical formalisms meaningful alternatives are possible. In the 1960's, Abraham Robinson developed the foundations of *non-standard analysis*, at the heart of which was an alternative formalization of the real-numbers in which points themselves have a deep structure. In

[17]Rucker, 1995, p. 82.

[18]Boyer, p. 564.

particular, it makes possible the resurrection of a version of *infinitessimals* that is logically precise. Unlike the standard real line, this non-standard real line is not unique, but may be constructed in a variety of ways. However, it always contains the standard real line embedded within it, and associated to each of the 'standard' points is associated a cloud of new points having a complex structure of its own. One way of imagining this is to think of the points on the continuum as consisting, not of discrete ordinates, but of infinitessimal regions, typically called *monads*, each fully as rich with structure as the entire standard set of real numbers.[19] Robinson's construction relies on advanced concepts in set theory,[20] and it may be objected on that account that the construction is *ad hoc* or artificial. But such an objection misses the point that our common conceptualization (Dedekind's) is equally dependent on set theoretical constructions and is similarly *ad hoc*.

On the basis of such possible conceptualizations, it is clear that we are free to consider the modern real numbers to be an imperfect *map* of time's continuum, but no more. Specifically, I suggest that what we call 'moments of time' are not dimensionless points at all; they are frameworks fully capable of embodying the dynamics that give rise to time and change. In short, time is textured. This is admittedly hard to picture, but then there

[19]Monads should not be thought of, however, as tiny copies of the real line, for typically their structure is far more complex.

[20]See for example Lengyel, 1996.

is no *a priori* reason to expect the nature of time to be so simple as the traditional stick-figure conception would have it. We are called upon to set aside our Euclidean notion of 'point' as 'that which has no parts,' and replace it instead with the very intuitions that led Newton and Leibniz to develop the calculus, which, as so many freshman learn to their wonderment each autumn, is the very science of change, and arguably the most successful, effective, and far-reaching intellectual achievement in human history. No longer is change constrained to abide within the dimensionless; infinitessimal change can and must occur in infinitessimal time, and the large scale flow of happenings which we seem to witness is merely the aggregate of these processes.

This idea of textured time should be carefully distinguished from certain other suggestions that have been put forth in recent years. In particular, the idea of merely compounding the points of the real line by adding new dimensions or by replacing them with 'tiny copies' of real spaces is to be avoided. This proposal should also be distinguished from 2-dimensional 'subjective' time, such as that suggested by Elliott Jaques,[21] for the structure I suggest for time is neither '2-dimensional' nor merely subjective. Neither does it amount to meta-time, as proposed for instance by George Schlesinger.[22] Instead, by textured time I intend that 'moments' be conceived neither as space-like entities of the sort typically modelled by

[21] Jaques, 1982.

[22] Schlesinger, 1980.

a standard real variable, nor as psychological artifacts, but rather as a new category of object in the world, one which demands and deserves metaphysical investigation.

How we are to form a positive conception, one capable of metaphysical investigation, of what I have here only adumbrated is by no means clear. It is not even clear whether an examination of mathematical structures (as possible models of time's texture) is helpful or harmful to this effort. In particular, I wish to emphasize that I do not construe Robinson's non-standard model of the continuum to be the correct model for time; its present value is to force the recognition that the notion of textured time may be susceptible to formalization and is not mathematically inconsistent – briefly, that such a conception is intellectually *possible*. The real impetus for our investigation is provided by the paradoxes: Zeno and McTaggart, between them, have demonstrated that without some such conception we must abandon our given, universal intuitions about time and change altogether, leaving us in an unintelligible world where our most basic perceptions are the blankest illusion.

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